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Stefan Folster & Georgi Trofimov

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Abstract: Many countries have reformed, or plan to reform, their pension system. The trend is to move from entitlement based systems to systems in which contributions accumulate in some form of personal savings account. In recent years proposals have been made to apply personal savings accounts also to other elements of social insurance, such as unemployment insurance, social assistance, health care costs and others. This paper analyzes the introduction of extended personal savings accounts in an overlapping generations model. The savings accounts provide both unemployment provision and saving toward retirement. The analysis shows under which conditions a savings account based social insurance can be designed to provide economic security at lower marginal tax rates than traditional social insurance.

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Appendix

1. Introduction

In many countries reforms of retirement income policy are discussed or actually implemented. A common trend is to move from public, pay-as-you-go schemes with defined benefits toward funded systems in which contributions are defined and deposited in some form of personal savings account, and benefits depend on the return to the assets invested. Often these systems also allow a larger role for private insurance companies.

In recent years there has been a growing interest in whether personal savings accounts can be a useful way of organizing other social insurance or social expenditure than just the retirement income system. The Singaporean Central Provident Fund, for example, was originally designed to increase savings and to provide retirement security. But it has since been extended with a number of schemes, e.g. saving for medical needs, financing of higher education, insurance of dependents and a variety of other social needs.[1] Chile is in the process of introducing an unemployment insurance based on a personal savings account. Consequences of introducing unemployment accounts have also been analyzed empirically by Feldstein and Altman (1998). Individual health accounts have been proposed and analyzed.[2] Proposals for more comprehensive savings account based reforms have been argued (e.g. Snower & Orszag, 1997; Fölster, 1997). In Fölster (1999) life cycle simulations, comparing current social insurance in Sweden with a savings account based system, indicate that it may be possible to combine economic security and income distribution with a significantly lower marginal tax rate.

The basic idea of more comprehensive savings account based systems is that mandatory payments into a personal savings account replace taxes currently used to finance unemployment benefits, sickness benefits, parental leave, pensions and other social insurances. When the need arises people are allowed to withdraw from their account instead of receiving benefits. At retirement the balance on the account is converted into an annuity that determines the pension level. Various insurance elements provide protection for those who deplete their account or suffer unexpected losses of life-time income.

The point of such a reform hinges on the insight that life-time income tends to be much more equally distributed than income in any particular year. As a result systems that use taxes to equalize income each year require higher tax rates than systems that aim to equalize life-time income. A savings account based social insurance provides a means for individuals to redistribute income between periods of life, thus reducing the need for tax-financed redistribution between periods of life.

[1] See Asher (1994) for a description of the the Singaporean Central Provident Fund.
[2] See for example Eichner et al. (1996).

The intuition behind this argument is illustrated in table 1 below. Assume an economy with two individuals. For simplicity their working life is divided into two periods only. Individual A has zero income in the first period, and individual B has zero income in the second period. Within each period their incomes are therefore characterized by a high degree of inequality. A conventional tax-based social insurance system would require a tax of 50% of income in each period to achieve full equalization, assuming that all revenue is transferred to the person with the lowest income in that period. Aggregate income throughout working life, however, is much more evenly distributed. This feature of the example is consistent with results from empirical studies.[3]

In an account-based social insurance system individuals would use an account to shift income between periods. The account is then insured, so that they receive compensation after the second period that supplements life-time income. Assume that this insurance is financed by a tax of 20 percent on current income, and the tax revenue is distributed to the person with the lower life-time income. Clearly this leads to exactly the same outcome. Life-time incomes are also fully equalized. Yet as long as each individual takes the tax- and subsidy rate as given, marginal taxes, and thus incentives, are dramatically lower in the account system.

Table 1. A simple example

	Individual A	Individual B
Income before tax, age 20-40	0	60
Income before tax, age 41-60	100	0
Total working-life Income, age 20-60 before tax	100	60
Tax rate required for full equalization in tax-based social insurance	50%	50%

3 Examples of such studies are Björklund (1989), and Aaberge et al. (1996). Eichner, McClellan and Wise (1996) show that even health care costs are much more evenly distributed across people in a life time perspective than is often believed.

Tax rate required for full equalization in account based insurance	20%	20%

The point illustrated in table 1 can also be described in terms of insurance theory. To the extent that social insurance deals with insurable events, both tax-financed transfers and actuarial insurance face a dilemma. Payment of the tax or insurance premium - as well as payment of the benefit - must be conditioned on declared income, which gives rise to a moral hazard problem. At least some individuals are able to abuse the transfer system or insurance by earning undeclared income or reducing their earnings below what they would be in first best optimum. For example, individuals may have lower incentives to invest in human capital. The presence of moral hazard implies that even an actuarial insurance gives rise to disincentives that are equivalent to marginal effects of taxes and subsidies. Unemployment compensation, for example, reduces incentives to earn income regardless of whether the compensation is provided by an actuarial insurance or a tax financed system.

Moral hazard can be addressed by introducing a deductible. The size of the deductible is limited, however, by welfare states´ desire to ensure the individual of a certain standard of living. Thus for people with a wage close to the acceptable minimum standard the deductible is effectively constrained to zero.[4]

Social insurance based on a personal savings account addresses this problem by using the account to shift premium payments and deductibles from periods where the individual has low income to other time periods during which the individual may have greater incentives or ability to earn higher income. As a result, the savings account allows a greater deductible than standard actuarial insurance, without compromising the standard of living in low-income periods or for individuals who always have low incomes.[5]

A crucial concern in evaluating such reforms and proposals is that they may be inequitable. Social insurance based on personal savings has often been viewed as incompatible with the aims of welfare states, partly because countries like Singapore that have savings account

[4] Even user fees charged for subsidized public services such as child care and public health care can be seen as a deductible in the presence of moral hazard. Fees cannot be raised much without putting the services out of the reach of poor people. If fees are related to income, then marginal taxes increase and the poverty trap problem is exacerbated. In a savings account system fees can be raised considerably, thus reducing overconsumption. The fees would be paid over the account which means that people can always afford the service even during low-income periods

[5] This mechanism will not work if all individuals either are always poor, or always have high incomes. Studies indicate, however, that income variability is considerable in European welfare states. An OECD study (Employment Outlook, july 1996) shows e.g. that half of the people in the lowest income quintile in Britain in 1986 had moved to a higher income group by 1991.

based systems provide very little redistribution. To the extent that redistribution or insurance is added to a savings account based system, one might be concerned that the gains in terms of lower marginal taxes are diluted.

In order to deal with these questions an overlapping generations model is analyzed here. The central mechanism is the same as in the example above. But in the model insurance rules, investment behavior and social welfare functions are explicitly developed. Within this framework we model a combined unemployment- and pension scheme that guarantees a minimum pension to those whose own income falls below a certain level, and compare this to an account based pension insurance.

In a first step a model is developed with only one period of uncertain income. In this type of model the insurance provided by the pension system can be made more efficient by the introduction of a savings account only if the account system is funded and earns a higher return than the tax-based pension system.

In a second step we extend the model to include two periods of varying income. This implies that, as in the simple example above, life-time income can be more evenly distributed than income in each period. This gives rise to the result that an insurance of life-time income may under some circumstances be more efficient than traditional social insurance of income within each period.

2. A model of social insurance

Consider an overlapping generation economy populated by initially homogenous individuals living two periods of time. Each generation has the same number of individuals, normalized to unity. The first period of individuals' lives is the period where they choose to invest in human capital such as education, training and on-the-job experience. One can think of the first period roughly as age 20-40. The stylized assumptions are made that individuals have equal income in this period, normalized to unity.

One can think of the second period of life as roughly the period beyond age 40. In the second period the human capital investment yields an uncertain return. The uncertainty reflects probabilities of disability or unemployment as well as uncertain future demand for the specific human capital. The social insurance we model aims to provide for some of these risks. In this section we do not distinguish between unemployment insurance, sickness benefits, and early retirement insurance. Instead we treat all of these as one social insurance that applies in the second period of life.

The return on human capital R is a non-negative stochastic variable with a continuously differentiable distribution function $F(R)$ having a measure support $[0,\infty)$. Individuals with return on human capital R earn second period primary income Rh, where h is individual investment in human capital. One can also interpret h more widely than merely human capital, e.g. to include savings more generally. In this case R also reflects the return to savings and even actuarial retirement savings regardless of whether this is voluntary or mandatory as part of a social insurance scheme.

Most public social insurance schemes contain considerable redistribution. We assume a two-tier insurance scheme that is common in many countries.[6] First, the tax-financed social insurance guarantees a minimum standard of living z in the second period of life. In addition it is assumed that in the second period of life each person receives a tax-financed retirement benefit, $\lambda h'$, proportional to average human capital investment h'. This reflects the ambition in most social security systems to let benefits increase in line with economic growth. Higher average investment in human capital will then increase primary second period incomes and retirement benefits.[7]

Since individuals are homogenous, $h' = h$ (prime is used in the following when necessary to make the distinction between individual and average variables). For simplicity coefficient λ is assumed to be an exogenous constant parameter. Since we focus only on the stationary regimes, the time subscripts are omitted henceforth, if it is not misleading.

The individual born in period t maximizes the expected utility of life-time consumption

$$\max_{h,c_1,c_2\geq 0} u(c_1)+\beta Eu(c_2), \tag{1}$$

subject to the budget constraints

$$c_1+h+\tau\leq 1, \tag{2}$$

$$c_2\leq y, \tag{3}$$

where c_1 is the first period consumption, c_2 and y are the second period consumption and disposable income, β is the discount factor. All individuals pay tax τ on the initial income in the first period of life. The budget constraints (2) and (3) relate to the first and second periods of life, correspondingly. Consumption utility $u(c)$ is monotonous, continuously differentiable and strictly concave.

Redistribution of income in this system is intergenerational: a proportion of income of the young is transferred to the old, but the transfer is not equally large to all old people. The social

[6] In Sweden, for example there is a base pension provided to everyone. In addition there is a sizable means-tested housing allowance for old people. In some countries there is no generally provided base pension. In our model that would be captured by the case where $\lambda = 0$.

insurance system guarantees that the individual disposable income in the second period of life is above the insured level z:

$$y = \max(Rh + \lambda h', z). \tag{4}$$

Figure 1 illustrates this constraint. A return R below a threshold return R_0 qualifies the individual for the pension guarantee. The insured income is calculated from (4) as

$$z = R_0 h + \lambda h'.$$

Given a threshold return $R_{0,}$ the insured income varies with h, so a higher investment provides a higher level of insured income. A return below R_0 entitles a person to a transfer $z - Rh - \lambda h' = (R_0 - R)h$.

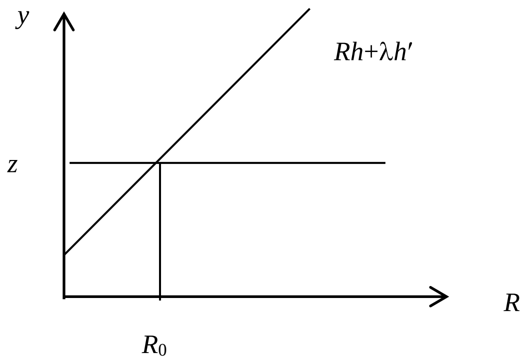

Figure 1

The individual decision problem (1)-(4) transforms to:

$$\max_h u(1-\tau-h) + \beta\int_{R_0}^{\infty} u(Rh+\lambda h')dF(R) + \beta u(z)F(R_0)\,. \tag{5}$$

The first order condition to this problem is

$$u'(1-\tau-h) = \beta\int_{R_0}^{\infty} Ru'(Rh+\lambda h')dF(R) + \beta R_0 u'(z)F(R_0)\,, \tag{6}$$

given that investment is positive. As is shown further below, this is the case in equilibrium.

We assume that the government is a benevolent policy maker who moves simultaneously with individuals to determine social insurance in the second period. The government weights utilities of concurrent generations equally in its objective function. This is the discounted sum of expected utilities $\sum_{t=0}^{\infty}\beta^t U_t$, where U_t represents at time t the life-cycle expected utility of the generation born in period t:

[7] We could assume that retirement benefit is individually defined as λh. It would not basically change the model results.

$$U_t = u(c_{1t}) + \beta Eu(c_{2t}) .$$

At time t+1 U_t represents the second-period weighted utility for the same generation:

$$U_t = Eu(c_{2t}),$$

where the symbol of expectation stands for averaging across old individuals. At each period of time the government redistributes income between concurrent generations, and its optimization problem therefore implies a step-by-step maximization of the one-shot weighted utility: $u(c_{1t}) + Eu(c_{2t-1})$. This is the sum of the first period consumption utility of the young and the weighted average utility of second period consumption by the currently old. The government sets the tax rate τ and the threshold return R_0.[8] Together with individual choice of investment, threshold return determines the insured income z.

In each period the government budget constraint balances total revenue and expenditures: [9]

$$\tau = (R_0 F(R_0) - \int_0^{R_0} RdF(R))h' + \lambda h' .$$

Government expenditures consist of the sum of guaranteeing the minimum income to those with low returns, and providing the base pension to all members of the old generation.

At any time period the policy problem is:

$$\max_{\tau, R_0} u(1-\tau-h) + \int_{R_0}^{\infty} u(Rh + \lambda h')dF(R) + u(z)F(R_0) , \qquad (7)$$

subject to the balanced budget equation

$$\tau = (g(R_0) + \lambda)h' \qquad (8)$$

where $g(R_0) = R_0 F(R_0) - \int_0^{R_0} RdF(R) = \int_0^{R_0} F(R)dR$. The function $g(R)$ is a cumulative distribution function satisfying the following properties: $g'(R) = F(R) \geq 0$, $g''(R) = F'(R) \geq 0$ for $R \geq 0$, $g(0) = 0$, and $g'(R)$ converges to 1 as R tends to infinity.

Proposition 1. Social insurance equalizes the first period consumption and the second-period insured income,

$$c_1 = z. \qquad (9)$$

[8] We could alternatively assume that the government chooses insured income instead of threshold return. This modification slightly complicates computations but does not crucially change the further analysis.

[9] There is no lending or borrowing by the government in the stationary regime.

Proof. Substituting (8) into the objective function (7), the first order condition for the policy problem is derived as

$$u'(1-\tau-h)g'(R_0)h' = u'(R_0h+\lambda h')F(R_0)h \tag{10}$$

The objective function (7) is concave in R_0, hence the first order condition is sufficient. Since $g'(R_0) = F(R_0)$, (10) implies that $u'(c_1) = u'(z)$ or $c_1 = z$.

Q.E.D.

Proposition 1 implies an intuitively appealing equalization of first-period consumption and insured income of young and currently old, which is a consequence of the assumption that taxes are paid only by the young and that the government equally weights utilities of concurrent generations. A marginal increase of threshold return causes a marginal increase of tax by $g'(R_0)h'$. This, in turn, causes a marginal decrease of first period utility by $u'(c_1)F(R_0)h'$. On the other hand, the marginal increase of second period average utility is $u'(z)F(R_0)h'$.

Given a threshold return R_0, Proposition 1 determines the level of human capital investment. Indeed, (10) implies that $1-\tau-h = (R_0 + \lambda)h$, or from (8)

$$h = 1/(1 + 2\lambda + R_0 + g(R_0)), \tag{11}$$

According to (11), the individual's propensity to invest is inversely related to the rate of retirement benefit λ and the threshold return R_0. Thus a more generous social insurance reduces incentives to invest.

3. Personal accounts

In this section we introduce a savings account which replaces the purely redistributive pension system. As will become clear this leads to a different outcome only due to the fact that retirement system is funded. No additional benefits of introducing an account emerge until the next section when variation in income is introduced in both periods.

In the first period individuals make mandatory transfers into accounts providing them with retirement benefits in the second period. Suppose that the account earns a return β^{-1}, that is the inverse of the discount factor. For ease of comparability assume further that the government guarantees a λ, which determines the base pension, at the same level as in the tax-based system. Then the face value of the account is $a = \lambda h'$, and the first period transfer into the account is $\beta\lambda h'$.

In the fully funded retirement system there is still redistribution providing social insurance benefits to those members of the old generation who earn a return below R_0. The social insurance transfer is $\tau = g(R_0)h'$. The individual decision problem modifies to:

$$\max_h u(1-\tau-\beta a-h)+\beta\int_{R_0}^{\infty} u(Rh+a)dF(R)+\beta u(z)F(R_0) \qquad (12)$$

where, as above, $z = R_0h + a = R_0h + \lambda h'$.

The government chooses a threshold return and a tax rate maximizing

$$u(1-\tau-\beta a-h)+\int_{R_0}^{\infty} u(Rh+a)dF(R)+u(z)F(R_0) \qquad (13)$$

subject to the balanced budget constraint

$$\tau = g(R_0)h'. \qquad (14)$$

The first order conditions of the individual and the government problems are

$$u'(c_1)=\beta\int_{R_0}^{\infty} Ru'(Rh+a)dF(R)+\beta R_0u'(z)F(R_0) \qquad (15)$$

which, in fact, is identical to (6), and

$$g'(R_0)u'(c_1)h' = u'(z)F(R_0)h. \qquad (16)$$

which is equivalent to:

$$c_1 = z. \qquad (17)$$

For any threshold return R_0, human capital investment is found from (17) and (14):

$$h = 1/(1 + (1 + \beta)\lambda + R_0 + g(R_0)), \qquad (18)$$

This equation differs from (11) which showed the propensity to invest under the tax-based system. If the threshold returns are the same in both systems, investment is higher under the account system. This is seen from comparison of (11) and (18): the term containing λ in (11) is lower because the account earns a return and this increases investment.

Given a threshold return, higher investment implies a higher insured income and a higher life-cycle expected utility, since $c_1 = z$, and $\partial[Eu(c_2)]/\partial z > 0$. The analysis of equilibria in the next section demonstrates that threshold returns are indeed identical in the funded and the tax-based systems. This means that all the welfare gain from introducing a funded personal account as compared to the tax-based system arises from the accumulation of interest on the account, which in turn increases incentives to invest.[10]

[10] We could easily incorporate a positive feedback from investment to wage into the model. This would imply an additional positive welfare effect of the account system resulting from interest earnings and higher investment. Another additional welfare effect would come from the assumption that social insurance under account system is also fully funded and accumulates on the generation account. This would completely eliminate intergenerational redistribution of income, and the one-shot objective function of the government at time period t would be U_t, the life-time expected utility of the young generation. The balanced budget constraint (14) then becomes $\tau = \beta g(R_0)h'$,

Once the model is extended to two periods of varying income, in section 5, this result is changed. In that case an account can reduce marginal taxes not just due to funding, but also due to the fact that, as discussed in the introduction, life-time income tends to be more equally distributed than income in each period.

4. Equilibria

In both the funded and the tax-based systems investment *h* is determined as a function of the threshold return as (11) or (18) indicates. Combining first order conditions related to individual choices and to government problems ((6) with (9) or (15) with (17)) yields the following equilibrium equation, identical for both systems:

$$u'(R_0 h + \lambda h') = \beta \int_{R_0}^{\infty} R u'(Rh + \lambda h') dF(R) + \beta R_0 u'(R_0 h + \lambda h') F(R_0) \qquad (19)$$

Suppose that consumption utility is isoelastic:

$$u(c) = \frac{c^{1-b} - 1}{1-b} \qquad (20)$$

and b>0. Then variable *h* vanishes from both sides of (19), which becomes a scalar equation on R_0:

$$(R_0 + \lambda)^{-b} = \phi(R_0) \qquad (21)$$

where $\phi(R_0) = \beta \int_{R_0}^{\infty} R(R+\lambda)^{-b} dF(R) + \beta R_0 (R_0 + \lambda)^{-b} F(R_0)$.

In what follows we assume that the base pension $\lambda h'$ is not excessively large, satisfying[11]

$$\lambda < \phi(0)^{-1/b}. \qquad (22)$$

Proposition 2. There is a unique equilibrium threshold return R_0 if $b \le 1$.*

Proof. Consider equation (21). The left-hand side is decreasing in R_0. The right hand side is increasing for all R_0 if $b \le 1$:

$$\phi'(R_0) = \beta (R_0 + \lambda)^{-b} F(R_0) - \beta b R_0 (R_0 + \lambda)^{-b-1} F(R_0) = \beta R_0 (R_0 + \lambda)^{-b} F(R_0) \left(1 - b \frac{R_0}{R_0 + \lambda}\right) > 0$$

Hence, because of (22), there is a unique equilibrium threshold return R_0*. Q.E.D.

but social insurance condition remains the same as (17). Investment function modifies to $h = 1/(1 + (1 + \beta)\lambda + R_0 + \beta g(R_0))$ exceeding (18) for a given value of R_0.

Figure 2a illustrates proposition 2. If b > 1, the function $\phi(R_0)$ is humped and there may be multiple equilibria[12]. Figure 2b demonstrates the case of two equilibrium threshold returns: R_0* and R_0**.

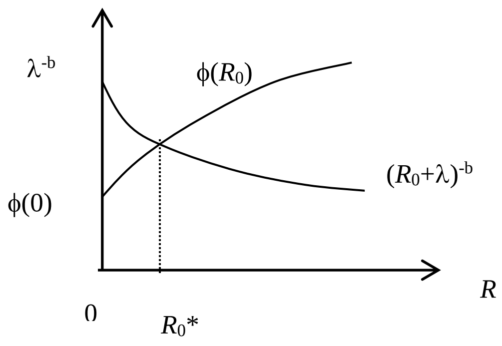

Figure 2a

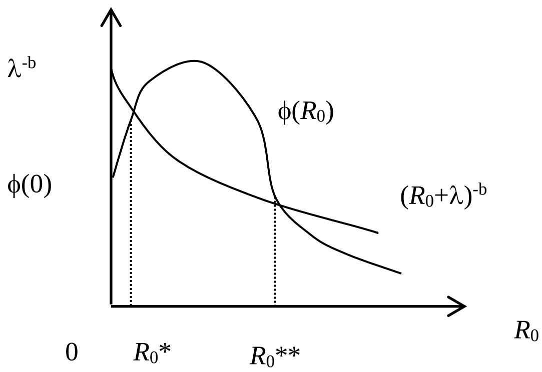

Figure 2b

Consequently, the equilibrium is unique if individual risk aversion is not too high. If it is

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[11] In fact, (22) is equivalent to: $\int_0^{\infty} R(1+R/\lambda)^{-b}\,dF(R) < 1$, hence it does not hold for large λ.

high, equilibria may arise with higher threshold returns and lower investment.[13] If the retirement benefit parameter λ is sufficiently large, then under the tax-based system insured income and welfare are higher in the high-marginal return equilibrium. This is not so for the funded system if $F(R_0^*) > \beta$, and a switch to the high-marginal return equilibrium reduces insured income[14].

5. Model with income variation in both periods

In this section the model is extended, assuming that individuals may experience a spell of unemployment in the first period of life. It is only in such a model with varying income in multiple periods that the main point of an account-based social insurance becomes apparent.

A share π of young people is unemployed and earns zero income in the first period. The share of unemployed is assumed to be fixed, and the same in the tax-based and the account system. With an endogenous unemployment rate the account system would imply a lower unemployment rate, since moral hazard is reduced.

Unemployed people receive unemployment benefits. For simplicity we assume that the unemployed do not pay taxes.[15] In the first period employed individuals receive a wage normalized to 1 and pay taxes that cover the base pension, the means-tested social insurance and unemployment benefits. The social insurance now contains both inter- and intragenerational redistribution of income. The second-period return on human capital is R with a cumulative density $F(R)$, identical for both groups.

The individual decision problem is to maximize a two-period expected utility:

$$u(c_{1i}) + \beta E u(c_{2i}) \tag{23}$$

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[12] The sufficient condition for multiple equilibria is $\phi(R_0') > (R_0' + \lambda)^{-b}$, where R_0' is the maximum point of $\phi(R_0)$, and $\lim_{R_0 \to \infty} \frac{(R_0 + \lambda)^{-b}}{\phi(R_0)} = 0$, i.e. distribution density $F'(R_0)$ tends to zero quit fast as R_0 goes to infinity (see fig. 2b).

[13] Multiple equilibria arise due to an external effect of investment on retirement benefit $\lambda h'$. If investment were decided by a social planner internalizing this effect, the optimal threshold return would be below R_0^*.

[14] Indeed, from (11) for the tax-based system $\frac{\partial z}{\partial R_0} = \frac{\partial((R_0 + \lambda)h)}{\partial R_0} = \partial\left(\frac{R_0 + \lambda}{1 + 2\lambda + R_0 + g(R_0)}\right) / \partial R_0 = \frac{1 + \lambda(1 - F(R_0)) + g(R_0) - R_0 g'(R_0)}{(1 + 2\lambda + R_0 + g(R_0))^2} > 0$ if λ is large enough.

For the funded system $\frac{\partial z}{\partial R_0} = \frac{1 + \lambda(\beta - F(R_0)) + g(R_0) - R_0 g'(R_0)}{(1 + (1+\beta)\lambda + R_0 + g(R_0))^2} < 0$ if $F(R_0) > \beta$ and λ is large. Since $F'(R_0) \geq 0$, $F(R_0) > \beta$, then for all $R_0 > R_0^*$ if $F(R_0^*) > \beta$.

[15] One could equivalently assume that they receive unemployment benefits net-of-tax.

subject to the budget constraints

$$c_{1i} + h_i \le y_{1i} \tag{24}$$

$$c_{2i} \le y_{2i} \tag{25}$$

where y_{1i}, y_{2i} are disposable incomes in periods 1 and 2, subscript i = e, u, relates to the employment status of the individual in the first period: 'e' means employed and 'u' - unemployed.

Unemployed young people receive unemployment benefits w_0 from the state. Disposable incomes of young individuals are $y_{1e} = 1-\tau$ and $y_{1u} = w_0$. Both types invest in human capital and earn a second period return Rh_i, i = e,u.

It is assumed that the unemployment period does not qualify individuals for the base pension.[16] As in the previous section the base pension is proportional to investment by the employed and equals $\lambda h_e'$. The unemployed receives a means-tested social insurance.

Disposable income in period 2 is determined from the familiar social insurance constraints

$$y_{2e} = \max(Rh_e + \lambda h_e', z_e), \tag{26}$$

$$y_{2u} = \max(Rh_u, z_u), \tag{27}$$

Insured second-period income could be assumed equal for both unemployed and employed since this is the case in many actual pension systems. But we consider the more general case where z_i is allowed to differ among employed and unemployed. The threshold return R_{0i} is also different for these groups. In fact, equalization of insured incomes results from optimization by the government, as shown further below.

The government moves simultaneously with individuals. It maximizes the discounted weighted sum of expected utilities $\sum_{t=0}^{\infty} \beta^t \left(\pi U_{tu} + (1-\pi) U_{te}\right)$, where, as above, U_{ti} is the expected life-cycle utility of the young person born in time t, or the weighted second-period utility of the old. The government determines the size of inter- and intragenerational redistribution as a solution to the policy problem, similar to (7), (8):

$$\max\left[\pi u(c_{1u}) + (1-\pi)u(c_{1e}) + \pi Eu(c_{2u}) + (1-\pi)Eu(c_{2e})\right], \tag{29}$$

subject to the balanced budget constraint

$$(1-\pi)\tau = (1-\pi)(g(R_{0e}) + \lambda)h_e' + \pi(g(R_{0u})h_u' + w_0)\,. \tag{30}$$

The control variables to the policy problem are τ, R_{0e}, R_{0u}, w_0.

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16 One could also assume that both employed and unemployed receive the base pension. This leads to qualitively similar conclusions, but complicates the computation somewhat.

As above, the symbol of expectation in the objective function (29) implies averaging between members of the old generation. The left-hand side of the budget constraint is total revenue. The right-hand side indicates the sum of retirement benefits to the employed, social insurance provision to members of the old generation and total unemployment benefits received by the young unemployed.

Proposition 3. The tax-based system equalizes first period consumption and insured income of both groups:

$$c_{1e} = c_{1u} \,, \tag{31}$$

$$c_{1e} = z_e \,, \tag{32}$$

$$c_{1e} = z_u \,. \tag{33}$$

Proof: in Appendix.

First period consumption is equalized by the unemployment insurance in the form of an intragenerational redistribution of income. Social insurance provides equalization of first period consumption and insured income in the second period.

As in the basic model, investment functions are found directly from social insurance conditions (31)-(33):

$$h_e = 1/\left[1 + 2\lambda + R_{0e} + g(R_{0e}) + \frac{\pi}{1-\pi}(\lambda + R_{0e})\left(1 + \frac{1 + g(R_{0u})}{R_{0u}}\right)\right], \tag{34}$$

and

$$h_u = \frac{\lambda + R_{0e}}{R_{0u}} h_e \tag{35}$$

Equations (34)-(35) are derived in the appendix. Given that there is no unemployment, $\pi = 0$, the investment function for the employed is identical to (11) derived for the basic model.

6. Personal accounts in the extended model

The account system allows unemployed to withdraw money from the accounts during unemployment. This system completely eliminates unemployment insurance and intragenerational redistribution in the first period. Personal accounts are opened at the beginning of life to finance retirement benefits. Instead of getting unemployment benefit the unemployed individual withdraws or borrows a certain amount of money d from the account. Transfers from the state are provided only to old individuals with income below the insured level.

Disposable incomes in the first period are $y_{1e} = 1 - \tau - a\beta$, $y_{1u} = d$. Only employed individuals are able to save on the account. The second period face value of the account is proportional to average investment made by the employed

$$a_e = \lambda h_e'.$$

An unemployed person borrows d from the account. If the unemployed person earns sufficient income in the second period, d and accumulated interest payments, $(\beta^{-1}-1)d$ are then repaid. If the second-period income is too low then the account is completely or partially redeemed by the social insurance system. We assume that an upper bound is imposed on withdrawals in order to prevent abuse of the account:

$$d \le d_u . \qquad (36)$$

For simplicity let d_u be an exogenous parameter.

The second period disposable incomes are

$$y_{2e} = \max((Rh_e + a_e, z_e),$$

$$y_{2u} = \max((Rh_u - d/\beta, z_u).$$

The threshold returns R_{0i} are determined from conditions $z_e = R_{0e}h_e + \lambda h_e'$ and $z_u = R_{0u}h_u - d/\beta$. The old recipient who was employed and earned the second-period return R receives social insurance $z_e - Rh_e - a_e = (R_{0e}-R)h_e$. Similarly, the old recipient who was unemployed receives social transfer $z_u - Rh_u + d/\beta = (R_{0u}-R)h_u$. Total second-period transfer to employed and unemployed members of the old generation is $g(R_{0e})h_e'$ and $g(R_{0u})h_u'$, respectively.

The individual decision problem of the employed is to select the consumption-investment bundle (c_{1e}, c_{2e}, h_e) solving (23)-(25). The problem of the unemployed is to choose the bundle (c_{1u}, c_{2u}, h_u, d) providing a maximum to (23)-(25) subject to the constraint on withdrawal (36).

The policy problem in period t is to maximize (29) subject to the balanced budget constraint

$$(1-\pi)\tau = (1-\pi)g(R_{0e})h_e' + \pi g(R_{0u})h_u' . \qquad (37)$$

The control variables are τ, R_{0e}, R_{0u}. The right hand side of (37) excludes retirement benefits, financed from the accounts by unemployed. Importantly, social insurance benefits and compensation for deficits on the account are not routinely paid to all who were unemployed in the first period of life, but only to those with income in the second period below the insured level z_u.

Proposition 4. The social insurance conditions under account system are:

$$c_{1u} = d_u - h_u\,, \qquad (38)$$

$$c_{1e} = z_e\,, \qquad (39)$$

$$c_{1e} = z_u\,. \qquad (40)$$

Proof: in the appendix.

Condition (38) means that the constraint on withdrawal (36) is binding, and d = d$_u$. *This is so, because individuals are insured against negative second-period incomes, and the life-time expected utility is increasing with withdrawal. The account-based system implies equalization of insured incomes for both groups, but does not guarantee equalization of the first-period consumption.*

The investment function of the employed is determined from (37) and (39) as

$$h_e = \frac{1 - \dfrac{\pi}{1-\pi} g(R_{0u}) h_u'}{1 + (1+\beta)\lambda + R_{0e} + g(R_{0e})}\,, \qquad (41)$$

If the share of unemployed is zero, the investment function of the employed is the same as (18), derived for the case of account system in the basic model.

7. Equilibrium in the extended model

Investment under the tax-based system are determined for both types by equations (34)-(35), given equilibrium threshold returns R_{0i}, i = e,u. As in the basic model these returns are found separately from investment in the case of isoelastic utility. Under the account system investment is determined for the employed in the same way as under the tax-based system. In fact, in both systems the first order condition for individual choice by the employed is the same as (6) or (15). The social insurance condition $c_{1e} = z_e$ is also the same for both systems, as stated by Propositions 3 and 4. These two conditions yield an equilibrium equation similar to (19)

$$u'(z_e) = \beta \int_{R_{0e}}^{\infty} R u'(Rh_e + \lambda h_e') dF(R) + \beta R_0 u'(z_e) F(R_0)\,. \qquad (42)$$

For isoelastic utility (20) equation (42) transforms into equation on R_{0e} similar to (21) and identical for both systems:

$$(R_{0e} + \lambda)^{-b} = \phi(R_{0e})\,. \qquad (43)$$

Hence, in equilibrium threshold return for the employed R_{0e} is the same in both systems. From Proposition 2, it exists and is unique for $b \le 1$. To simplify the matter, we assume in the following that $b \le 1$.

Consider individual choices by the unemployed. For the tax-based system the first-order condition related to h_u is

$$u'(w_0 - h_u) = \beta \int_{R_{0u}}^{\infty} R u'(R h_u) dF(R) + \beta R_{0u} u'(z_u) F(R_{0u}) .$$

This is combined with social insurance conditions (31), (33) linking c_{1u} and z_u. Since $z_u = R_{0u}h_u$, these conditions yield:

$$R_{0u}{}^{-b} = \phi_1(R_{0u}) \tag{44}$$

where $\phi_1(R_{0u}) = \beta \int_{R_{0u}}^{\infty} R^{1-b} dF(R) + \beta R_{0u}{}^{1-b} F(R_{0u})$. The left-hand side of (44) is decreasing in R_{0u}, and the right-hand side is non-decreasing (for $b \le 1$). An equilibrium return for the unemployed always exists and is unique.

One can easily find the exact solution to (44) if the utility function is logarithmic, that is $b = 1$:

$$R_{0u}^{\tau} = \beta^{-1}. \tag{45}$$

In this case the threshold return for the unemployed equals the riskless return. Here and henceforth the superscript τ refers to the tax-based system, and the superscript a to the account system.

Consider equilibrium conditions for the unemployed person under the account system. He makes positive investment, otherwise $y_{2u} < 0$. Because of (38), the individual choice condition related to h_u is

$$\begin{aligned} &u'(d_u - h_u) = \\ &\beta \int_{R_{0u}}^{\infty} R u'(R h_u - d_u / \beta) dF(R) + \beta R_{0u} u'(R_{0u} h_u - d_u / \beta) F(R_{0u}). \end{aligned} \tag{46}$$

For isoelastic utility this yields

$$h_u = s(R_{0u}) d_u, \tag{47}$$

where $s(R_{0u})$ is a propensity of the unemployed to invest, $0<s(R_{0u})<1$. The equilibrium threshold return under account system is then determined from (47) and social insurance conditions (39)-(40) implying that $z_u = z_e$:

$$R_{0u} s(R_{0u}) = \beta^{-1} + (R_{0e} + \lambda) h_e / d_u. \tag{48}$$

This equation has a unique solution, $R_{0u}{}^{a}$, if its left-hand side is monotonously increasing and $s(R_{0u})$ is separated away from 0 for all R_{0u} . In this case an equilibrium exists and unique for both systems. Given the equilibrium threshold returns ($R_{0e}{}^{\tau}$, $R_{0u}{}^{\tau}$) or ($R_{0e}{}^{a}$, $R_{0u}{}^{a}$), investment is determined for each group through investment functions (34), (35) or (41), (47).

8. Welfare comparisons

To compare welfare in both systems we assume that the upper limit imposed on withdrawal equals unemployment benefits paid in equilibrium under the tax-based system, that is

$$d_u = w_0. \qquad (49)$$

Then the question we address is under what conditions the account system improves welfare of all groups.

Welfare comparison is based on the social insurance conditions formulated in propositions 3 and 4. Higher investment by the employed yields higher welfare for both groups. Indeed, as above, $\partial[Eu(c_{2i})]/\partial z_i > 0$, for i = e, u. Under both systems the employed person's first period consumption equals his insured income, $c_{1e} = z_e = \lambda h_e + R_{0e} h_e'$. The threshold return of the employed, R_{0e}, is the same in both systems. Hence the life-cycle expected utility of the employed is higher under the system where his investment is higher.

Due to equalization of insured incomes stated by proposition 3, the first period consumption and insured income of the unemployed under the tax-based system equal $z_e{}^{\tau} = (\lambda+R_{0e})h_e{}^{\tau}$. Then his welfare is also increasing with the level of investment decided by the employed. Due to (50), the first-period income of the unemployed is the same under both systems. His second-period insured income under the account system equals $z_e{}^{a} = (\lambda+R_{0e})h_e{}^{a}$, because of proposition 4. Hence, the life-cycle utility of the unemployed will be higher under the account system if $z_e{}^{a} > z_e{}^{\tau}$, that is investment by the employed is higher: $h_e{}^{a} > h_e{}^{\tau}$.

As a result, the first period consumption and insured income of both groups are determined by the employed person's investment. If investment is higher in one of the systems, welfare will be higher for both groups. Consequently, conclusions about welfare in the different systems must be based on comparison of investment functions of the employed (34) and (41).

First, as in the basic model, individuals under the account system pay less into the account than is needed to finance the equivalent retirement benefit under the tax-based system. This is captured by the second term in the denominator of investment functions (34), (41) related to retirement benefits provision. This term is equal to 2λ for the tax-based system and $(1+\beta)\lambda$ for the account-based system. The difference arises from the interest accumulated on the account.

Second, the account system can provide a diminished intragenerational redistribution of income. Employed people do not finance unemployment benefits. They compensate withdrawals only for the share of unemployed unable to receive sufficient income in the second period of life. However, insurance payments even to this group may be high when the equilibrium threshold return $R_{0u}{}^{a}$ is high. The resulting effect of diminished intragenerational redistribution on investment depends on how much unemployed individuals are in fact compensated by the state at old age.

Proposition 5. The account system welfare-dominates the tax-based system if

$$1+\frac{g(R_{0u}^{\tau})}{1+R_{0u}^{\tau}} > g(R_{0u}^{a})s(R_{0u}^{a}). \tag{50}$$

Proof: in the appendix.

Given (50), the account system is welfare-dominating because $h_e{}^{a} > h_e{}^{\tau}$. In fact, proposition 5 ensures that the account system is superior in terms of social insurance provision. This is so, because (50) is equivalent to $\tilde{h}_e^a > h_e^\tau$, where $\tilde{h}_e^a$ is investment by the employed (41) with term $(1+\beta)\lambda$ in the denominator substituted by 2λ, and $\tilde{h}_e^a < h_e^a$. If (50) holds, investment by the employed remains higher under the account system even if one completely eliminates a positive effect of interest accumulation on the account.

This shows that in a model with income variation in both periods the possible efficiency gain in an account system is not only due to funding. Rather, the life-cycle principle embodied in the account system can provide a welfare gain through a more efficient social insurance. [17] It should be noted that the model is cautious in the sense that no dynamic comsequenes of lower marginal effects in the account system on unemployment are considered.

We can suggest a simple and intuitively appealing approximation for (50) if we consider the case of logarithmic utility and a small unemployment rate.

Proposition 6. Let b = 1,

$$1 + \beta > g(\beta^{-1}), \tag{51}$$

[17] Additional welfare effects of the account system would result from a positive feedback of investment on wage, and from introduction of fully funded social insurance completely eliminating intergenerational redistribution (see footnote 10).

and

$$\frac{\pi}{1-\pi} \le \frac{\beta}{2(1+\beta-g(\beta^{-1}))}. \tag{52}$$

Then the account system welfare-dominates the tax-based system if

$$1+\beta \ge [g(\mu(\beta)) - g(\beta^{-1})]\beta, \tag{53}$$

where

$$\mu(\beta)=\begin{cases}\beta^{-2}+2.5\beta^{-1} & if\ \ \beta \ge 0.5\\ \beta^{-2}+2\beta^{-1}+\dfrac{\beta^{-1}-0.5}{1+\beta} & if\ \ \beta < 0.5.\end{cases}$$

Proof: in Appendix.

Condition (51) rules out discount factors that are too low to fulfil (50) for any values of model parameters π and λ.[18] Hence, (51) is a necessary condition for (50). Condition (52) imposes an upper bound on the unemployment rate. Conditions (52)-(53) are sufficient for (50).

One can easily show that the right-hand side of (53) is decreasing with β.[19] Therefore, (53) is fulfilled if the discount factor is sufficiently high or, given any discount factor, if $g(\mu(\beta))$-$g(\beta^{-1})$ is small. The latter condition depends on the properties of the distribution function $F(R)$.

Consider a class of distributions represented as $F(R,\theta)$, where θ is a positive parameter and $\partial F(R,\theta)/\partial\theta \le 0$ for all positive R. Let $F(R,\theta)$ converge to 0 as θ tends to infinity. Distributions belonging to this class are ordered in the sense that a higher θ implies a higher mean return: if $\theta'' > \theta'$ then $ER(\theta'') \ge ER(\theta')$ where $ER(\theta)$ stands for the mean for $F(R,\theta)$.[20] Then $g(\mu(\beta))$-$g(\beta^{-1})$ is decreasing with θ since $\mu(\beta) > \beta^{-1}$ and $\partial^2 g(R,\theta)/\partial\theta\,\partial R = \partial F(R,\theta)/\partial\theta \le 0$. For this class of distributions (53) holds if θ and, hence, the mean return is sufficiently large.

As an example consider an exponential distribution $F(R,\theta) = 1 - e^{-R/\theta}$. Then

$$g(R,\theta) = R-\theta(1-e^{-R/\theta}),$$

the mean return is θ, $\partial F(R,\theta)/\partial\theta \le 0$ for all R, and (53) may be written as

$$2+\beta \ge \left(\mu(\beta)+\theta(e^{-\mu(\beta)/\theta}-e^{-1/\theta\beta})\right)\beta. \tag{54}$$

Given any discount factor β, the mean return θ must be quite high in order for (54) to hold.

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[18] This is seen from (50) and expression (A19) in Appendix.

[19] It is sufficient to show that $d[(g(\beta^{-2})-g(\beta^{-1}))\beta]/d\beta < 0$. This is so because $g(R)$ has increasing returns to scale.

[20] Indeed, $ER(\theta) = \int_0^\infty [1-F(R,\theta)]dR$ (see Feller 1966, p. 190), hence, $ER(\theta'') \ge ER(\theta')$ if $F(R,\theta'') < F(R,\theta')$ for all R.

Table 2 shows the relation between the discount factor (the riskless return) ranking between 0.5 and 0.9, and the critical level of the mean return, θ_a, above which (54) fulfils and the account system is preferable. As seen from the table, the higher the discount factor is, the lower is the mean return that satisfies (54). Table 2 also shows another cririical level of the mean return, θ_τ, below which the necessary condition (51) does not hold. If $\theta < \theta_\tau$, the tax-based system is prefarable to the account system.

Table 2. The critical values of the mean return.

β	0.5	0.6	0.7	0.8	0.9
$1/\beta$	2	1.67	1.43	1.25	1.11
θ_a	9.44	5.84	3.84	2.62	1.82
θ_τ	0.51	0.07	–	–	–

To get some intuition of whether these conditions are fulfilled in actual life one could reason as follows. An annual real discount rate of 2 percent would yield a discount factor of close to 0.6 over a 25-year period. This might be a reasonable approximation of the time that elapses from the middle of the first period of life in the model to the middle of the second period. The rate of return on human capital could be calculated as the non-discounted, before-tax return of a university education. In Sweden such an approximation to θ_a would take a value of slightly above 6 or 7.4 percent per annum. With these approximated values an account system would clearly fulfil the criteria established in table 2, and therefore be socially preferable.

Here it should be remembered the conditions of proposition 6 implemented in table 2 for θ_a are sufficient conditions. If the conditions are not fulfilled the account system may still be better, but this remains ambiguous. The tax-based system is preferable for quite low mean returns on human capital, as is seen from the last raw in table 2. The symbol "–" in the table means that θ_τ does not exist at all, that is the necessary condition (51) fulfils for all positive θ. In fact, this is the case if $\beta > 0.618$. In addition the social benefits of reduced moral hazard in unemployment insurance in the account system are not explicitly considered.

Table 3 demonstrates the upper bounds on unemployment rate π_0 turning the constraint (52) into equality for different discount rates and mean returns. The constraint imposed on π

seems empirically relevant, because the long-run rates of unemployment in most developed countries are below these levels.

Table 3. Upper bound for the unemployment rate that fulfills the constraint (52)

β	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*
π_0(%) for θ = 2	*24.6*	*22.0*	*21.3*	*21.3*	*21.5*
π_0(%) for θ = 4	*18.9*	*18.8*	*19.2*	*19.8*	*20.3*
π_0(%) for θ = 6	*17.2*	*17.8*	*18.5*	*19.2*	*20.0*

The account system is thus preferable to the tax-based system if either the riskless return is small or the mean return on human capital investment is large. A practical implication of this result is that social insurance based on the account system should be introduced under a proper economic environment characterized with high human capital income or low interest rate.

The discount factor has an ambiguous effect on welfare gains due to saving accounts. On the one hand, a higher discount rate provides gains through accumulation of interest earnings on these accounts and diminished taxation. On the other hand, a lower discount rate ensures a positive welfare effect from reducing social insurance transfers under the account system. Since retirement provision under this system is superior for any interest rate, a net positive welfare effect from applying the extended personal accounts is guaranteed if the interest rate is low.

Conclusion

Most countries already have some element of social insurance based on a mandatory savings account. Pension systems and student loans often work this way. A number of countries, among them Sweden, have recently reformed their pension systems, moving from an entitlement system to a savings account based system. [21] In a number of countries savings account based systems are also under consideration for training of both employed and unemployed. "Individual Learning Accounts" were, for example, proposed by the British Labour Party. [22]

For other types of social insurance savings accounts are less common. One example, however, is the Chilean unemployment insurance. Newly employed are there required to save in the form of monthly installments until savings reach a value of two months' wages. If a person becomes unemployed the savings are paid back over a four month period. Only after that public assistance steps in. Saved funds follow employees if they change employer. At retirement saved funds are paid out. In essence the scheme creates a larger deductible, but helps to spread the impact over a longer time period.

The analysis in this paper showed that an account system may increase efficiency in social insurance, beyond what can be explained by the effect of funding the account. This effect arises in models with income variation in at least two periods. For plausible ranges of parameter values the account system is socially preferable to a tax-based system. Social insurance can, however, be modelled in many different ways. Future research should determine how robust these results are in different model specifications.

[21] A smaller part of contributions in the new system will be channeled into real savings accounts, while the larger part continues to work on the pay-as-you-go principle. In essence bookkeeping accounts are built up that reflect a drawing right on future generations' payments. Individuals will have some choice as to how the real savings are to be invested.
[22] in "New Deal for a Lost Generation", presented May 15th, 1996.

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Appendix

Proof of Proposition 3.

Consider the government problem (29)-(30). Inserting τ from budget constraint (30) into the argument c_{1e} in the objective function (29), and differentiating it with respect to R_{0e} yields a first order condition similar to (11):

$$(1-\pi)g'(R_{0e})u'(1-\tau-h_e)h_e' = (1-\pi)u'(R_{0e}h_e+\lambda h_e')F(R_{0e})h_e' \qquad \text{(A1)}$$

Because $g'(R_{0e}) = F(R_{0e})$, (A1) is equivalent to

$$c_{1e} = z_e \qquad \text{(A2)}$$

Similarly, the first-order condition related to R_{0u} is

$$(1-\pi)(\pi/(1-\pi))g'(R_{0u})u'(1-\tau-h_e)h_u' = \pi u'((R_{0u}h_u+\lambda h_u')F(R_{0u})h_u'$$

or

$$c_{1e} = z_u. \qquad \text{(A3)}$$

The first order condition related to unemployment benefit w_0 is

$$u'(c_{1e}) = u'(c_{1u}),$$

that is

$$c_{1e} = c_{1u}. \qquad \text{(A4)}$$

Conditions (A2), (A3), and (A4) are stated as proposition 3.

Q.E.D.

Derivation of equations (34), (35).

From (31)-(33) we obtain $z_u = z_e$ or

$$(R_{0e}+\lambda)h_e = R_{0u}h_u .$$

This yields (35).

From (31) and (35) we obtain

$$w_0 = c_{1e} + h_u = (R_{0e}+\lambda)h_e + h_u = (R_{0e}+\lambda)(1+1/R_{0u})h_e .$$

Inserting w_0 and (35) into the government budget constraint (30) yields the tax rate:

$$\tau = \left[g(R_{0e}) + \lambda + \frac{\pi}{1-\pi}\left(R_{0e}+\lambda\right)\frac{1+g(R_{0u})+R_{0u}}{R_{0u}} \right] h_e$$

Equation (32) means that

$$1 - \tau - h_e = (R_{0e}+\lambda)h_e$$

Inserting τ in this equation implies (34).

Proof of Proposition 4.

Consider the government problem (29), (37). Plugging τ from (37) into c_{1e} in the objective function (29), yields the first order condition for R_{0e}:

$$(1-\pi)g'(R_{0e})u'(c_{1e})h'_e = (1-\pi)u'(z_e)F(R_{0e})h'_e \qquad \text{(A5)}$$

which is equivalent to

$$c_{1e} = z_e \qquad \text{(A6)}$$

The first-order condition for R_{0u} is

$$(1-\pi)(\pi/(1-\pi))g'(R_{0u})h'_u u'(1-\tau-h_e) = \pi u'(R_{0u}h_u - d/\beta)F(R_{0u})h'_u$$

or

$$u'(c_{1e}) = u'(z_u)$$

which yields $c_{1e} = z_u$.

Consider the individual decision problem (23)-(25), (36). Suppose the constraint on withdrawal (36) is not binding. Then the individual first order condition on d is

$$u'(c_{1u}) = \int_{R_{0u}}^{\infty} u'(Rh_u - d/\beta)dF(R) + u'(R_{0u}h_u - d/\beta)F(R_{0u}) + \zeta, \qquad \text{(A7)}$$

where ζ is a Lagrange multiplier related to (36). The first order condition for h_u is

$$u'(c_{1u}) = \beta\int_{R_{0u}}^{\infty} Ru'(Rh_u - d/\beta)dF(R) + \beta R_{0u}u'(R_{0u}h_u - d/\beta)F(R_{0u}). \qquad \text{(A8)}$$

Since $c_{1u} = d - h_u > 0$, $d > h_u$. Since $z_u = R_{0u}h_u - d/\beta > 0$, then

$$R_{0u} > \beta^{-1}. \qquad \text{(A9)}$$

Subtracting both sides of (A7) from (A8) we obtain

$$\zeta = \int_{R_{0u}}^{\infty} (\beta R-1)u'(Rh_u - d/\beta)dF(R) + (\beta R_{0u}-1)u'(z_{0u})F(R_{0u}). \qquad \text{(A10)}$$

Both terms on the right-hand side of (A10) are positive, because of (A9). Hence, $\zeta > 0$ and an upper constraint on d is binding. This yields (38).

Q.E.D.

Proof of Proposition 5.

According to (41)

$$h_e^a = \frac{1-\dfrac{\pi}{1-\pi}g(R_{0u}^a)h_u'^a}{1+(1+\beta)\lambda + R_{0e} + g(R_{0e})} > \frac{1-\dfrac{\pi}{1-\pi}g(R_{0u}^a)h_u'^a}{1+2\lambda + R_{0e} + g(R_{0e})} = \tilde{h}_e^a \qquad \text{(A11)}$$

We omit superscript of the system for R_{0e} since it is the same in both systems.

Applying subsequently conditions (47), (49), (31), (32), (35) we transform h'^a_u:

$$h'^a_u = s(R^a_{0u})d_u = s(R^a_{0u})w_0 = s(R^a_{0u})(c^\tau_{1e} + h^\tau_u) = s(R^a_{0u})(z^\tau_e + h^\tau_u) = s(R^a_{0u})\big((R_{0e}+\lambda) + (R_{0e}+\lambda)/R^\tau_{ou}\big)h^\tau_e = s(R^a_{0u})(R_{0e}+\lambda)(1+1/R^\tau_{ou})h^\tau_e \quad \text{(A12)}$$

Substituting h'^a_u into (A11) yields

$$\tilde{h}^a_e = \frac{1 - \frac{\pi}{1-\pi} g(R^a_{0u})s(R^a_{0u})(1+1/R^\tau_{0u})(R_{0e}+\lambda)h^\tau_e}{1+2\lambda+R_{0e}+g(R_{0e})}$$

Then condition

$$\tilde{h}^a_e > h^\tau_e \quad \text{(A13)}$$

is equivalent to

$$\frac{1/h^\tau_e - \frac{\pi}{1-\pi} g(R^a_{0u})s(R^a_{0u})(1+1/R^\tau_{0u})(R_{0e}+\lambda)}{1+2\lambda+R_{0e}+g(R_{0e})} > 1$$

Taking into account (34) this can be written as

$$1 + \frac{\frac{\pi}{1-\pi}\left(1+\frac{1+g(R^\tau_{0u})}{R^\tau_{0u}}\right)(\lambda+R_{0e}) - \frac{\pi}{1-\pi} g(R^a_{0u})s(R^a_{0u})(1+1/R^\tau_{0u})(\lambda+R_{0e})}{1+2\lambda+R_{0e}+g(R_{0e})} > 1$$ or,

equivalently,

$$1+\frac{1+g(R^\tau_{0u})}{R^\tau_{0u}} > g(R^a_{0u})s(R^a_{0u})\left(1+\frac{1}{R^\tau_{0u}}\right).$$

This yields (50).

Q.E.D.

Proof of Proposition 6.

The proof consists of several steps.

Step 1. First we show that for logarithmic utility

$$s(R_{0u}^{a}) > \beta/(1+\beta) \qquad \text{(A14)}$$

Indeed, applying the implicit function theorem to (46) yields

$$\frac{ds}{dR_{0u}} = \frac{\beta(R_{0u}s-\beta^{-1})^{-1}F(R_{0u})+\beta R_{0u}s(R_{0u}s-\beta^{-1})^{-2}F(R_{0u})}{(1-s)^{-2}+\beta\int_{R_{0u}}^{\infty}R^2(Rs-\beta^{-1})^{-2}dF(R)+\beta R_{0u}^2(R_{0u}s-\beta^{-1})^{-2}F(R_{0u})} =$$

$$\frac{\beta(R_{0u}s-\beta^{-1})^{-2}F(R_{0u})((R_{0u}s-\beta^{-1})-R_{0u}s)}{(1-s)^{-2}+\beta\int_{R_{0u}}^{\infty}R^2(Rs-\beta^{-1})^{-2}dF(R)+\beta R_{0u}^2(R_{0u}s-\beta^{-1})^{-2}F(R_{0u})} =$$

$$-\frac{(R_{0u}s-\beta^{-1})^{-2}F(R_{0u})}{(1-s)^{-2}+\beta\int_{R_{0u}}^{\infty}R^2(Rs-\beta^{-1})^{-2}dF(R)+\beta R_{0u}^2(R_{0u}s-\beta^{-1})^{-2}F(R_{0u})} < 0,$$

because $R_{0u}s > \beta^{-1}$ from (48) (for notational briefness we dropped here the superscript a related to R_{0u}).

If R_{0u}^{a} tends to infinity, (46) transforms to

$$(1 - s)^{-1} = \beta s^{-1}.$$

Hence $s(R_{0u}^{a})$ converges to $\beta/(1+\beta)$ as R_{0u}^{a} tends to infinity. Then (A14) fulfils, because $s'(R_{0u}^{a}) < 0$ for all $R_{0u}^{a} > \beta^{-1}$.

Step 2. Then we find an upper bound for the right-hand side of (50). From (48), (49), (A12), (45)

$$R_{0u}^{a} = [\beta^{-1} +(R_{0e} + \lambda)h_e^{a}/d_u]/s(R_{0u}^{a}) = [\beta^{-1} +(R_{0e} + \lambda)h_e^{a}/(R_{0e} + \lambda)(1+1/R_{0u}^{\tau})h_e^{\tau}]/s(R_{0u}^{a}) = [\beta^{-1} + h_e^{a}/(1+\beta)h_e^{\tau}]/s(R_{0u}^{a}). \qquad \text{(A15)}$$

Since $g(R)$ has increasing marginal returns, and from (A14), (A15)

$$g(R_{0u}^{a})s(R_{0u}^{a}) = g[(\beta^{-1} + h_e^{a}/(1+\beta)h_e^{\tau})/s(R_{0u}^{a})]s(R_{0u}^{a}) < g[(\beta^{-1} + h_e^{a}/(1+\beta)h_e^{\tau})(1+\beta)/\beta]\beta/(1+\beta). \qquad \text{(A16)}$$

According to (45) $R_{0u}^{\tau} = \beta^{-1}$. Hence (51) fulfils if

$$1 + g(R_{0u}^{\tau})/(1+R_{0u}^{\tau}) =$$

$$1 + g(\beta^{-1})\beta/(1+\beta) \geq g[(\beta^{-1} + h_e^{a}/(1+\beta)h_e^{\tau})(1+\beta)/\beta]\beta/(1+\beta),$$

or

$$1 + \beta + g(\beta^{-1})\beta \geq g(\beta^{-2} + \beta^{-1} + \beta^{-1} h_e^a / h_e^\tau)\beta. \qquad \text{(A17)}$$

<u>Step 3.</u> Now we find an upper bound on h_e^a/h_e^τ. From (34), (41), (A12), (45)

$$\frac{h_e^a}{h_e^\tau} = \frac{1/h_e^\tau - \frac{\pi}{1-\pi} g(R_{0u}^a)s(R_{0u}^a)(1+1/R_{0u}^\tau)(R_{0e}+\lambda)}{1+(1+\beta)\lambda + R_{0e} + g(R_{0e})} =$$

$$1+\frac{(1-\beta)\lambda + \frac{\pi}{1-\pi}(R_{0e}+\lambda)(1+\beta)\left[1+\frac{g(\beta^{-1})\beta}{1+\beta} - g(R_{0u}^a)s(R_{0u}^a)\right]}{1+(1+\beta)\lambda + R_{0e} + g(R_{0e})} \qquad \text{(A18)}$$

Since $g(R_{0u}^a)$ has increasing returns to scale,

$$g(R_{0u}^a)s(R_{0u}^a) = g[(\beta^{-1} + h_e^a/(1+\beta)h_e^\tau)/s(R_{0u}^a)]s(R_{0u}^a) >$$

$$g[\beta^{-1} + h_e^a/(1+\beta)h_e^\tau] > g(\beta^{-1}).$$

Then

$$(1+\beta)\left(1+\frac{g(\beta^{-1})\beta}{1+\beta} - g(R_{0u}^a)s(R_{0u}^a)\right) < (1+\beta)\left(1+\frac{g(\beta^{-1})\beta}{1+\beta} - g(\beta^{-1})\right)$$
$$= 1+\beta - g(\beta^{-1}), \qquad \text{(A19)}$$

and (50) fulfils only if (51) holds.

Because of (52) we have from (A18), (A19)

$$\frac{h_e^a}{h_e^\tau} < 1+\frac{(1-\beta)\lambda + (R_{0e}+\lambda)\beta/2}{1+(1+\beta)\lambda + R_{0e} + g(R_{0e})} = 1+\frac{(1-\beta/2)\lambda + R_{0e}\beta/2}{1+(1+\beta)\lambda + R_{0e} + g(R_{0e})}$$
$$< 1+\frac{(1-\beta/2)\lambda + R_{0e}\beta/2}{(1+\beta)\lambda + R_{0e}} < 1+\max\left(\frac{\beta}{2}, \frac{1-\beta/2}{1+\beta}\right) < 1+\max\left(\frac{1}{2}, \frac{1-\beta/2}{1+\beta}\right). \qquad \text{(A20)}$$

<u>Step 4.</u>

Due to (A20), (A17) holds if

$$1 + \beta + \beta g(\beta^{-1}) \geq \beta g[\beta^{-2} + \beta^{-1}(2+\chi(\beta)]\beta,$$

where

$$\chi(\beta) = \max\left(\frac{1}{2}, \frac{1-\beta/2}{1+\beta}\right).$$

This is equivalent to (53).

Q.E.D.